Assessment Schedule – 2005

Scholarship Statistics and Modelling (93201)

Evidence Statement

As shown in the schedule below, either a nine point marking scale (0-8) or a seven point marking scale (0-6) were used to assess the questions.

QUESTION ONE (Total 22 marks)

Task: Q1 (a) (6 marks)

Evidence:

Let x = number of mince pies and y = number of steak pies.
Constraints are: $x + y \geq 42$, $x \geq 24$, $3y \geq x$ and $y \leq x + 12$.
By sketching the feasible region and minimising $C = 1.0x + 1.3y$ we get the **optimal point (31,11)**, ie 31 mince pies and 11 steak pies.

Judgement:

Optimal point is correctly calculated and identified.

Task: Q1 (b) (8 marks)

Evidence:

The cost per mince pie would need to increase to $1.30 each. The new optimal points become (24,18), (25,17), (26,16), (27,15), (28,14), (29,13), (30,12) and (31,11), ie coinciding with the line $x + y = 42$. Once the cost of a mince pie has increased beyond $1.30, the optimal solution becomes (24,18).

Judgement:

BOTH optimal solutions correctly identified, for the differing costs of a mince pie associated with all relevant point(s).

If only one optimal solution identified … (4 marks), must be clear about integer points, otherwise 0.

Task: Q1 (c) (8 marks)

Evidence:

$S^2 = 66M + 10$ cuts the edge of the feasible region at $y = x + 12$ so we need to solve the quadratic $x^2 - 42x + 134 = 0$. This gives rise to $x = 39$ (integer) and $y = 51$. This optimal solution **(39, 51)** satisfies all the constraints and minimises C.

Judgement:

Optimal order point is correctly calculated.

Other optimal points are (38,50) and (39,50) which give minimums of \$103 and \$104 respectively rather than \$105.30 These answers are acceptable.

QUESTION TWO (Total 28 marks)

Task: Q2 (a) 1 (6 marks)

Evidence:

Pr (wrongly identified as Type B) = Pr (weight > 24.9) = Pr ($Z > 1.917$) = 0.5 – 0.4723 = 0.0277 so **2.77%**

Judgement:

Percentage correctly determined.
If 0.0277 … (5 marks)

Task: Q2 (a) 2 (8 marks)

Evidence:

Pr (incorrectly identified) = Pr (incorrectly identified as Type A and is Type B)
+ Pr (incorrectly identified as Type B and is Type A)
Pr (incorrectly identified as Type A and is Type B) = Pr (weight < 24.9) = Pr ($Z < -2.3$)
= 0.5 – 0.4893 = **0.0107**
Pr (incorrectly identified) = 0.0107 + 0.0277 (answer to Q2 (a) 1) = **0.0384**

Pr (Type B given incorrectly identified) = 0.0107 ÷ 0.0384 = 0.279 so **27.9%**

Judgement:

Conditional **percentage** is correctly determined.
If 0.279 … (7 marks)
If error in one number eg 0.017 instead of 0.0107 (–1 mark) provided answer reasonable.
Other answers are: 28% (7 marks), 107/384 (7 marks) or ratio 2681/9591 (5 marks).
Full credit given if no further errors from Q2 (a) 1 being incorrect.

Task: Q2 (a) 3 (8 marks)

Evidence:

Weigh each bag of 20 and calculate the mean weight MW = total weight ÷ 20.
If MW < 24.9 classify as Type A, otherwise Type B. There would be virtually no incorrect identifications as the standard deviation is reduced to a standard error of $\frac{\sigma}{\sqrt{n}}$ = 0.27 for Type A and 0.22 for Type B. Hence no overlap in the two distributions of "the mean". This is illustrated by the following three sigma limits for each type: for type A we have $22.6 \pm 3 \times 0.27$ ie 21.79 to 23.41, and for type B we have $27.2 \pm 3 \times 0.22$ ie 26.54 to 27.86. Observe that there is no overlap.

Judgement:

Improvement clearly explained with justification incorporating the use of at least one calculation.
Could work with the total weight of 20 items as an alternative with a reasonable cut-off point being accepted like 498 g. Must have clear explanation.
Calculation with no explanation scores zero.
Mean of 452 and 544 not sufficient without a spread consideration (5.367 and 4.472 for the totals A and B respectively).

Task: Q2 (b) (6 marks)

Evidence:

$\lambda = 6$ over 30 minutes.
Want m such that $\text{pr}(x \leq m) = 0.95$ where x = number of arrivals in 30 minutes.
Note that $\text{pr}(x \leq 10) = 0.9575$ is the closest to 0.95.
Number of customer arrivals that would be exceeded in only 5% of half-hourly intervals, in the long run, is **10**.

Judgement:

Number of customer arrivals to meet criteria is correctly determined. Must show working.
Normal approximation scores zero, however no penalty if done as well.
Could have "exceeded nine" with working.
"Eleven" scores 4 marks.

QUESTION THREE (Total 26 marks)

Task: Q3 (a) (6 marks)

Evidence:

$\alpha = 0.95, \quad n = 20, \qquad \bar{x} = \dfrac{31.7385 + 24.9015}{2} = 28.32$

$e = 3.4185$

$\therefore \quad s = 7.8$

99% CI: (23.827,32.813) years

Judgement:

99% confidence interval correctly determined.
(23.8,32.8) is acceptable. Can use $z = 2.576$

Task: Q3 (b) (8 marks)

Evidence:

$z = 2.58$ (2.576 okay), $e = 2$ and $s = 7.8$

So $n = \left(\dfrac{2.58 \times 7.8}{2} \right)^2 = 101$, so select **81** more *Megabitz* customers.

Judgement (both required)

- Additional sample size is correctly determined.
- Assumption is clearly stated that the standard deviation in the larger sample is similar to the standard deviation of the sample of size 20. Can use $z = 2.576$.

If $n = 101$ … (7 marks).
No assumption with $n = 81$ … (6 marks).
No assumption with $n = 101$… (5 marks).
Note that assumption scores two marks.
No marks if $e = 1$.
If s is incorrect can get 6 marks if no other error.

Task: Q3 (c) (1) (6 marks)

Evidence:

Compute a confidence interval for the difference in two population means:
$\mu_1 - \mu_2$ where μ_1 = population mean male expenditure and
μ_2 = population mean female expenditure

Now $-1.11 - 1.96 \times 0.415 < \mu_1 - \mu_2 < -1.11 + 196 \times 0.415$

So, $-\$1.92 < \mu_1 - \mu_2 < -\$0.30$.

Judgement:

95% confidence interval correctly determined.
Also, $-\$1.90 < \mu_1 - \mu_2 < -\$0.30$ is acceptable.
No $ sign … (5 marks).
If not rounded … (5 marks).
Can have other way round.

Task: Q3 (c) (2) (6 marks)

Evidence:

Yes, the confidence interval calculated in 3(c)(i) doesn't contain zero, which implies that the mean expenditure of females is higher than that of males.

Judgement:

Correct conclusion that mean expenditure of females is significantly higher than that of males, with justification.
Answer with justification of a difference (5 marks).
Conditional on previous answer.

QUESTION FOUR (Total 46 marks)

Task: Q4 (a) (6 marks)

Evidence:

1. There is an increasing non-linear relationship between *B* and *C*.
2. Rate of increase in time spent using the refreshment bar over computer time is almost zero from $C = 0$ to125; reasonably constant from $C = 126$ to 350 and is increasing from $C = 351$ to 450. (Note limits in ranges for *C* are approximate only.) **OR**
3. Could be described as a piecewise model: Horizontal linear (B = constant approx. 20) for $C = (0,125)$, linear in the form $B = mC$ + constant for $C = (215,375)$ and vertical linear $C = 375$ (approx.).
4. Description in context: people who spend up to about two hours at the computers tend to use the bar for approximately 20 minutes; for people who spend between two hours and six hours at the computers the tendency is to use the refreshment bar more as time on computers increases; people seem to reach a maximum time of just over six hours on the computer, and then the time spent in the refreshment bar varies between 3 and 4 hours, thus spending approximately between 9 and 10 hours at *Megabitz* in a day. **OR**
5. The total time spent at *Megabitz* per person varies from approximately one hour to approximately 9–10 hours per day. (Computer time + time spent using the bar.)

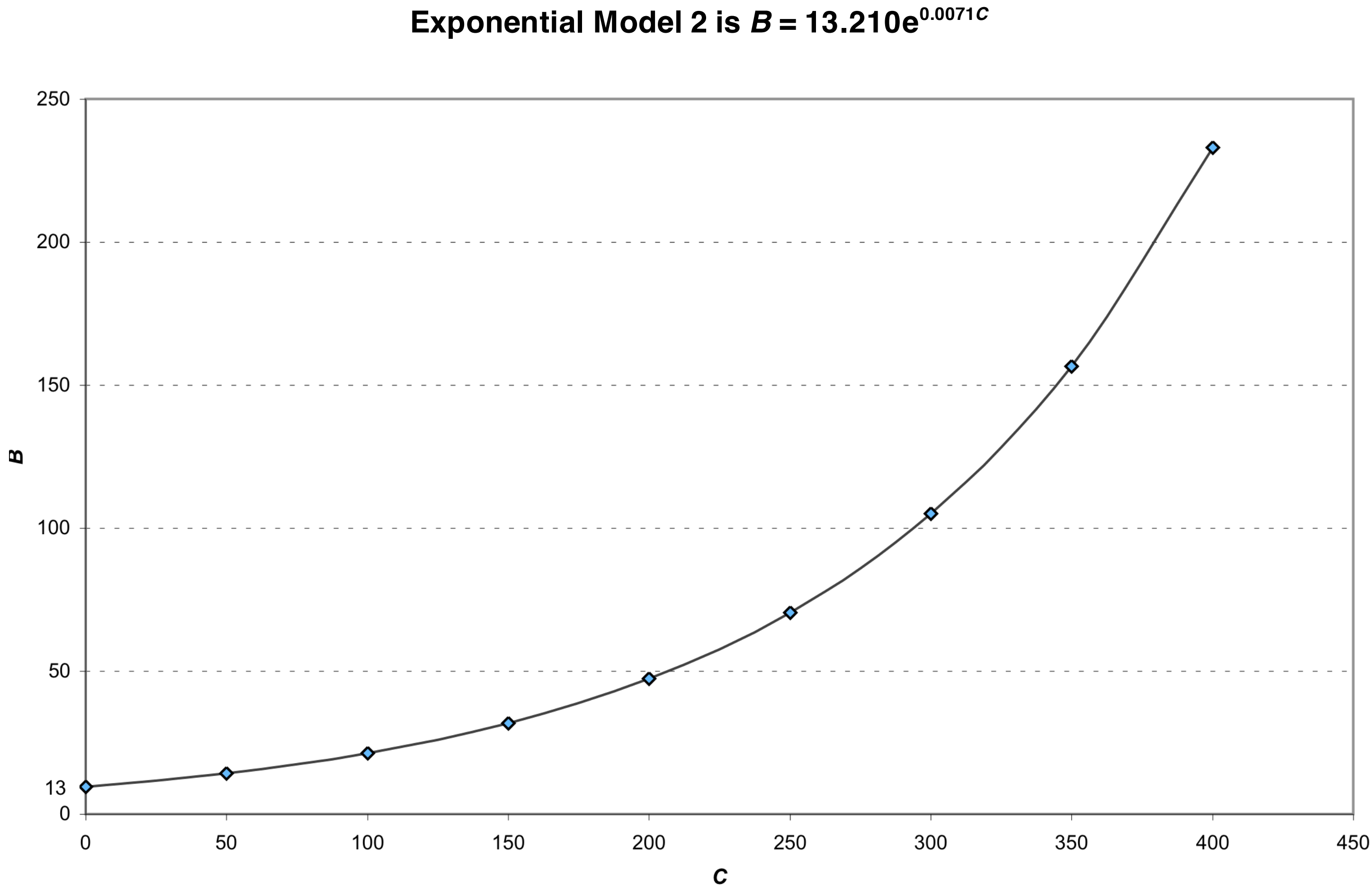
Judgement:

A description that mentions THREE distinct relevant comments (1; 2 or 3; 4 or 5) in context about the relationship.
Two points … (4 marks).
One point … (2 marks).
No credit for reference to model or R^2.

Task: Q4 (b) (6 marks)

Evidence:

$\ln B = 2.5810 + 0.0071C$



Judgement:

Sketch graph should show B axis intercept at (0,13) and a reasonable upwards curve from that point to an end-point in the approximate correct position to (400,226).
No labels (–1 mark).
13 not shown (–1 mark).
No scales (–2 marks).
If B isn't shown as the subject of the equation, then no marks.
Can have $B = 13.21 \times 1.007^{C}$ as an answer.

Task: Q4 (c) (6 marks)

Evidence:

The **exponential model** would be useful for predicting *B* for low values of *C*. When ***C* = 30 in the linear model, *B* = –19.901,** which is not a sensible answer since one cannot spend negative time using the refreshment bar. (Note *C*-intercept for Model 1 is 66.3). If $C < 66$ then *B* is negative in the linear model.

Judgement:

Exponential model with justification. Must have justification.
Not accepted that R^2 is higher for the exponential model.

Task: Q4 (d) (6 marks)

Evidence:

Possible answers (1dp):

C (min)	Model 1	Model 2
180	62.3 min	47.4 min
405	185.5 min	234.3 min

Judgement:

At least THREE predictions out of four worked out correctly using both models**.** Can have predictions to nearest whole number.
MEI: Variation in answers tolerated.
If only two predictions … (4 marks).
One prediction … (2 marks).
Units not penalised.

Task: Q4 (e) (8 marks)

Evidence:

For higher values of *C*, the linear Model 1 is quite removed from the actual dataset, while Model 2 (higher R^2) follows the actual values more closely. It is better, therefore, to use Model 2 to predict *B* for *C* = 405 minutes (6 h 45 minutes). A limitation for predicting beyond the actual dataset is that this is only valid if all factors influencing *C* and *B* remain the same and the relationship between *B* and *C* remains exponential defined as above.

For *C* = 3 hours, the values predicted by either model seem reasonable with respect to the data with *C* within the given data.

Judgement:

One acceptable suitability statement covering each *C* value with 4 marks, each giving rise to the predictions in (d).
"Just fits best" is not acceptable.

Task: Q4 (f) (6 marks)

Evidence:

A variety of methods are acceptable. Get the following:

C	Model 1	Model 2
124.8	32.02	32.04
124.9	32.08	32.07
344.2	152.19	152.15
344.3	152.24	152.26

Model 1 gives higher values when $124.9 \le C \le 344.2$

Note: A sketch would be helpful in ascertaining that the line (Model 1) is above the exponential curve (Model 2) within the overall range of *C* values.

Judgement:

BOTH *C* values correctly determined. 125 and 344 are acceptable limits.
One limit correct … (3 marks).
Graphics calculator gives 124.86638 and 344.28898.
If answer to 4(b) is $B = 13.21 \times 1.007^{C}$, then 123.4 and 356.7 are acceptable.

Task: Q4 (g) (8 marks)

Evidence:

- The exponential curve fits more closely to the actual data values for lower values of *C* and also for higher values of *C*.
- The more appropriate model is the exponential Model 2. This model with a log transformation of *B* has a higher R^2 value than the simple linear Model 1. ($R^2 = 0.9627$ compared with $R^2 = 0.8951$).
- The linear model is not valid for these data for values of *C* less than 66.3 because for all $C < 66.3$, $B < 0$. In context, there cannot be negative time spent using the refreshment bar.
- The exponential model is a reliable model for $0 < C < 400$. This model would have to be treated with caution for values of $C > 400$.
- Assuming a 24-hour restriction, for $C > 587$ the exponential Model 2 is not sensible since the total time prediction exceeds 24 hours (1440 min). This implies that the total time spent in *Megabitz* is 24 hours when $C = 587$ min.

Judgement:
Select Model 2 with at least THREE relevant points not already stated.
Two points … (5 marks).
One point … (3 marks).
Answer to Q4 (c) not to be repeated.

QUESTION FIVE (Total 14 marks)

Task: Q5 (a) (6 marks)

Evidence:

$14.8 = 400a + 20b + c$ ………..①

$20.6 = 900a + 30b + c$ …………②

$15.3 = 1600a + 40b + c$ …………③

Solution: $a = –0.0555$, $b = 3.355$, $c = –30.10$ all to **4 sf.**

Model: Expenditure $= –0.0555 \times (\text{age})^2 + 3.355 \times \text{age} – 30.10$

Judgement:

Equations correctly formed and solved. Correct rounding to 1 d.p. acceptable.
If model not stated and values of a, b and c correct (5 marks).
Okay to use x and y in the model equation.

Task: Q5 (b) (8 marks)

Evidence:

This is a good decision since the average expenditure by 50-yr-olds given by substituting into the model is –$1.10 at the café, which is negative, so it is not possible. The indication from the extrapolated model is that there would be minimal spending by 50-yr-olds at the café. Clearly *Megabitz* would not be able to make the return on the expense of setting up the lounge.

$$\begin{aligned}\text{Expenditure} &= –0.0555 \times (\text{age})^2 + 3.355 \times \text{age} – 30.10 \\ &= –0.0555 \times (50)^2 + 3.355 \times 50 – 30.10 \\ &= –\$1.10\end{aligned}$$

Average Expenditure = 0 when age = 48.9 years. Age > 48.9 $\Rightarrow$ model gives negative average expenditure.

Model has inapplicability for x values greater than 50 outside the range of the data (15–45 years).

Judgement:

A justification for the decision backed up by an appropriate calculation.
Correct appropriate calculation only (2 marks).
No follow through from (a) if quadratic has positive a value.

QUESTION SIX (Total 20 marks)

Task: Q6

Evidence:

Note: The report should cover the following points:

1. A comment on the overall pattern in the data. Must refer to fluctuations.
2. A comment related to the summary statistics provided for each six-month period. Must have both mean and standard deviation.
3. A reference to the sales peak.
4. A comment on the seasonal variation observed in the relationship between actual sales and CMA values.
5. A comment on what the CMA graph shows.
6. A comment on the fits of the line and curve to the CMA graph.
7. A calculated forecast for February 2006 by two methods, one using the line and the other the curve.
8. At least **two out of three** distinct comments on the usefulness and limitations of the forecasts.

Eg
Report:

Overall the sales have fluctuated between $12 000 and $20 000 approximately in the two years to Oct 2004. Over the last year since then the monthly sales have dropped steadily.

For each six-monthly interval the mean and the median of the monthly sales have been approximately equal. There has been little variation in the monthly sales over each six-monthly interval with the exception of the six months, May 2003 to Oct 2003, when the standard deviation virtually doubled.

Sales peaked in July 2004 at $29 000.

The most favourable months for sales where the actual sales exceeded the yearly monthly average were July, September, October and November. The least favourable months, when the reverse occurred, were January, February, March, May and August.

The graph of the yearly monthly averages, namely the CMA curve, peaked at February 2004 and has dropped ever since.

The curve provides an excellent fit to the CMA graph ($R^2 = 0.9527$) compared with the line that provides a moderate fit to the CMA curve ($R^2 = 0.2249$).

Forecast Calculations:

Seasonal Factor:

For February: $S = \dfrac{(15 - 18.25 + 13 - 13.54)}{2} = -1.895$

Using Trend Line:

$x = 34$, so Trend Value = –0.1112 × 34 + 17.709 = 13.928

Forecast = 13.928 – 1.895 = 12.033 ie **$12 000**

Using Curve:

$x = 34$, so Trend Value = –0.0324 × 34 × 34 + 0.6987 × 34 + 14.199 = 0.500

Forecast = 0.500 – 1.895 = –1.395 ie **–$1 400**

Comments on Forecasts:

The forecast obtained using the curve is negative so is clearly not possible. Clearly, the fitted curve would no longer apply in February 2006.

The forecast using the line with its negative gradient is suspect because we are forecasting 10 months ahead of when the fitted line finished.

For both models, the seasonal factor has been calculated over only two Februaries, giving two values that are vastly different. More Februaries would be required to give an established figure.

Judgement:

Points 1 to 6: One mark per point in context, with maximum 6 marks.

Point 7: Three marks per forecast.
If obtain 12.033 (2 marks), –1.395 (2 marks), no $000 (2 marks).
If wrong month with everything else correct (1 mark per forecast).
If no seasonal adjustment then no marks.

Point 8: THREE relevant comments covering limitations.
Two comments (5 marks).
One comment (3 marks).

Judgement Statement

An aggregate mark of 156 from six questions was used in Statistics and Modelling.
In 2005, candidates who achieved 107-156 marks were awarded outstanding scholarship and candidates who achieved 70-106 marks were awarded scholarship.